

# Testing the EoS of dark matter with cosmological observations

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We explore the cosmological constraints on the parameter  $w_{\text{dm}}$  of the dark matter barotropic equation of state (EoS) to investigate the “warmness” of the dark matter fluid. The model is composed by the dark matter and dark energy fluids in addition to the radiation and baryon components. We constrain the values of  $w_{\text{dm}}$  using the latest cosmological observations that measure the expansion history of the Universe. When  $w_{\text{dm}}$  is estimated together with the parameter  $w_{\text{de}}$  of the barotropic EoS of dark energy we found that the cosmological data favor a value of  $w_{\text{dm}} = 0.006 \pm 0.001$ , suggesting a *warm* dark matter, and  $w_{\text{de}} = -1.11 \pm 0.03$  that corresponds to a phantom dark energy, instead of favoring a cold dark matter and a cosmological constant ( $w_{\text{dm}} = 0, w_{\text{de}} = -1$ ). When  $w_{\text{dm}}$  is estimated alone but assuming  $w_{\text{de}} = -1, -1.1, -0.9$ , we found  $w_{\text{dm}} = 0.009 \pm 0.002, 0.006 \pm 0.002, 0.012 \pm 0.002$  respectively, where the errors are at  $3\sigma$  (99.73%), i.e.,  $w_{\text{dm}} > 0$  with at least 99.73% of confidence level. When  $(w_{\text{dm}}, \Omega_{\text{dm}0})$  are constrained together, the best fit to data corresponds to  $(w_{\text{dm}} = 0.005 \pm 0.001, \Omega_{\text{dm}0} = 0.223 \pm 0.008)$  and with the assumption of  $w_{\text{de}} = -1.1$  instead of a cosmological constant (i.e.,  $w_{\text{de}} = -1$ ). With these results we found evidence of  $w_{\text{dm}} > 0$  suggesting a *warm* dark matter, independent of the assumed value for  $w_{\text{de}}$ , but where values  $w_{\text{de}} < -1$  are preferred by the observations instead of the cosmological constant. These constraints on  $w_{\text{dm}}$  are consistent with perturbative analyses done in previous works.

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## I. INTRODUCTION

The astrophysical evidence for the existence of Dark Matter (DM) is well based on observations from the scales of galaxies, clusters and the universe itself, in the framework of the standard cosmological model.

Despite the fact that the cosmological scenario where cosmological parameters fit a dark matter mainly non-baryonic and cold, a great debate has currently opened about the possibility that Warm Dark Matter becomes a better candidate to understand the recent investigations. For a wide discussion based in the new results in the area see [1]. A summary of astrophysical constraints on dark matter is present in [2]

Let us summarize some of the difficulties of the Cold Dark Matter (CDM) model. At galactic scales, N-body simulations of cosmological structures with CDM have predicted that the dark matter halos surrounding galaxies must present radial profiles of the mass density and velocity dispersion with a central cusp in which the value of the logarithmic slope is under discussion (see [3]).

In the case of the *missing satellites problem* [4], [5] exists a discrepancy in the cold dark matter model between the predicted numbers of satellite galaxies inside the galactic halo for the Milky Way and lower number observed. This problem has been undertaken assuming a warm dark matter component in various works [6].

A recent investigation of N-body simulations in a warm dark matter scenario, which took into account the new dwarf spheroidal galaxies discovered in the Sloan Digital Sky Survey (SDSS) [7], derived lower limits on the dark matter particle mass [8].

In recent investigations, the measuring of the dark matter equation of state (EoS) has been carried out using different approaches. Following a suggestion given in [9], where the method combines kinematic and gravitational lensing data, the dark matter EoS was measured in [10] using galaxy clusters which present gravitational lensing effects. The result of this work indicates that the measured EoS for dark matter is consistent with the standard pressureless cold dark matter at  $1\sigma$  level. Nevertheless, lensing analysis in clusters such as Coma and CL0024 shows a trend to prefer an exotic EoS for the dark matter, i.e.,  $w \sim -1/3$ .

Models of the dark matter component described by a fluid with non-zero effective pressure has been studied in some astrophysical scenarios. At galactic level, an EoS with anisotropic pressures has been investigated in [11] in order to explain flat rotation curves. A polytropic dark matter halo fits very well a number of elliptical galaxies, improving or at least giving similar results to the

velocity dispersion profile compared to a stars-only model [11].

Explorations of the EoS for dark matter at cosmological level have been carried out in various frameworks. In [13], a constant EoS for dark matter is studied from the study of the power spectrum, assuming a cosmological constant as the dark energy fluid and a flat universe. The bounds obtained for  $w_{\text{dm}}$  were  $-1.50 \times 10^{-6} < w_{\text{dm}} < 1.13 \times 10^{-6}$  if there is no entropy production and  $-8.78 \times 10^{-3} < w_{\text{dm}} < 1.86 \times 10^{-3}$  if the adiabatic sound speed vanishes. Phenomenologically, EoS for both dark fluids have been studied in [14] using WMAP+BAO+ $H_O$  observations by synchronizing the model with the  $\Lambda$ CDM model at the present time. The dark matter component behaves like radiation at very early times and at the present time  $w_{\text{dm}} = 0.0005$ .

In the case of unified dark matter models, where a single matter component is assumed to source the acceleration and structure formation [15], the initial phase is described by a cold dark matter so that the fitting with cosmological data leads to a late phase with negative  $w_{\text{dm}}$  very close to a cosmological constant or phantom matter.

Our aim in this work is to study the EoS of the dark matter component allowing a non zero value for  $w_{\text{dm}}$  from the beginning and then to undertake a constraining of its value using the latest observations that measure the expansion history of the universe. In what follows, we will assume a barotropic EoS for this component. Of course, this assumption is rather restrictive because in approaches based in the nature of particles constituting the dark matter fluid is expected to have a  $w_{\text{dm}}$  varying with the cosmological time. Such is the case, for example, for dark matter Bose-Einstein condensation [16]. Nevertheless, if the dark matter fluid is modeled, in the non-relativistic approximation, as a non-degenerated ideal Maxwell-Boltzmann gas, a barotropic EoS is obtained with  $w_{\text{dm}} = \text{constant}$  [17].

The present paper is organized as follows. In Section II, we briefly outline the basic equations of evolution of the model. In Section III, the parameters of the model are constrained using cosmological data from type Ia supernovae, CMBR, baryon acoustic oscillations, the Hubble expansion rate and the age of the universe. Finally, in Section IV, we discuss and conclude our results.

## II. THE COSMOLOGICAL MODEL

We study a cosmological model composed by four fluids: radiation, baryons, dark matter and dark energy. We assume a barotropic equation of state (EoS) for dark matter (dm) and energy (de) fluids,  $p_i = w_i \cdot \rho_i$ , with  $i = \text{dm}, \text{de}$ , respectively.  $\rho_i$  corresponds to the density of the fluid and  $p_i$  to its pressure. We are interested in studying the cosmological prediction for the EoS of the

dark matter, in particular, for the magnitude of  $w_{\text{dm}}$ .

We assume a spatially flat Friedmann-Robertson-Walker (FRW) cosmology. The Friedmann constraint and conservation equations for the radiation, baryonic, dark matter and dark energy fluids are given respectively as

$$H^2 = \frac{8\pi G}{3} (\rho_{\text{r}} + \rho_{\text{b}} + \rho_{\text{de}} + \rho_{\text{dm}}) \quad (1)$$

$$0 = \dot{\rho}_{\text{r}} + 4H\rho_{\text{r}} \quad (2)$$

$$0 = \dot{\rho}_{\text{b}} + 3H\rho_{\text{b}} \quad (3)$$

$$0 = \dot{\rho}_{\text{dm}} + 3H\rho_{\text{dm}}(1 + w_{\text{dm}}) \quad (4)$$

$$0 = \dot{\rho}_{\text{de}} + 3H\rho_{\text{de}}(1 + w_{\text{de}}), \quad (5)$$

where  $H$  is the Hubble parameter and the dot over  $\dot{\rho}_i$  stands for the derivative with respect to the cosmic time. The conservation equations (2)-(4) have the respective solutions in terms of the scale factor  $a$

$$\rho_{\text{r}}(a) = \frac{\rho_{\text{r}0}}{a^4}, \quad \rho_{\text{b}}(a) = \frac{\rho_{\text{b}0}}{a^3}, \quad \rho_{\text{dm}}(a) = \frac{\rho_{\text{dm}0}}{a^{3(1+w_{\text{dm}})}}, \quad \rho_{\text{de}}(a) = \frac{\rho_{\text{de}0}}{a^{3(1+w_{\text{de}})}}, \quad (6)$$

where the subscript zero at  $\rho_{i0}$  indicates the present-day values of the respective matter-energy densities. Inserting the expression (6) on the Friedmann constraint (1), and dividing by the Hubble constant  $H_0$ , it becomes

$$E^2(a) \equiv \frac{H^2(a)}{H_0^2} = \frac{8\pi G}{3H_0^2} \left( \frac{\rho_{\text{r}0}}{a^4} + \frac{\rho_{\text{b}0}}{a^3} + \frac{\rho_{\text{dm}0}}{a^{3(1+w_{\text{dm}})}} + \frac{\rho_{\text{de}0}}{a^{3(1+w_{\text{de}})}} \right). \quad (7)$$

We define the dimensionless parameter densities as  $\Omega_{i0} \equiv \rho_{i0}/\rho_{\text{crit}}^0$ , where  $\rho_{\text{crit}}^0$  is the critical density evaluated today defined as  $\rho_{\text{crit}}^0 \equiv 3H_0^2/(8\pi G)$ . With this definition, the Friedmann equation (7) obtains the form

$$E^2(a) = \frac{\Omega_{\text{r}0}}{a^4} + \frac{\Omega_{\text{b}0}}{a^3} + \frac{\Omega_{\text{dm}0}}{a^{3(1+w_{\text{dm}})}} + \frac{\Omega_{\text{de}0}}{a^{3(1+w_{\text{de}})}}, \quad (8)$$

or, using the relation between the scale factor and the redshift “ $z$ ” given by  $a = 1/(1+z)$ , we rewrite the dimensionless eq. (8) in terms of the redshift as

$$E^2(z) = \Omega_{\text{r}0}(1+z)^4 + \Omega_{\text{b}0}(1+z)^3 + \Omega_{\text{dm}0}(1+z)^{3(1+w_{\text{dm}})} + \Omega_{\text{de}0}(1+z)^{3(1+w_{\text{de}})}. \quad (9)$$

Setting  $E(z = 0) = 1$  we have the constraint equation,

$$\Omega_{\text{de}0} = 1 - (\Omega_{\text{r}0} + \Omega_{\text{b}0} + \Omega_{\text{dm}0}). \quad (10)$$

### III. COSMOLOGICAL CONSTRAINTS

To constrain the value of  $w_{\text{dm}}$  using cosmological data, to compute their confidence intervals and to calculate their best estimated values, we use the following cosmological observations described below measuring the expansion history of the Universe.

To perform the numerical calculations, it was used for the baryonic and radiation (photons and relativistic neutrinos) components the values of  $\Omega_{\text{b}0} = 0.0458$  [19] and  $\Omega_{\text{r}0} = 0.0000766$  respectively, where the later value is computed from the expression [20]

$$\Omega_{\text{r}0} = \Omega_{\gamma 0}(1 + 0.2271N_{\text{eff}}) \quad (11)$$

where  $N_{\text{eff}} = 3.04$  is the number of standard neutrino species [19, 21] and  $\Omega_{\gamma 0} = 2.469 \times 10^{-5}h^{-2}$  corresponds to the present-day photon density parameter for a temperature of  $T_{\text{cmb}} = 2.725$  K [19], where  $h$  is the dimensionless Hubble constant  $h \equiv H_0/(100 \text{ km/s}\cdot\text{Mpc})$ .

#### 1. Type Ia Supernovae

We use the type Ia supernovae (SNe Ia) of the “Union2.1” data set (2012) from the Supernova Cosmology Project (SCP) composed of 580 SNe Ia [22]. The luminosity distance  $d_L$  in a spatially flat FRW Universe is defined as

$$d_L(z, w_{\text{dm}}) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z', w_{\text{dm}})} \quad (12)$$

where “ $c$ ” corresponds to the speed of light in units of km/sec. The theoretical distance moduli  $\mu^t$  for the  $k$ -th supernova at a distance  $z_k$  is given by

$$\mu^t(z, w_{\text{dm}}) = 5 \log \left[ \frac{d_L(z, w_{\text{dm}})}{\text{Mpc}} \right] + 25 \quad (13)$$

So, the  $\chi^2$  function for the SNe Ia test is defined as

Best estimates for $(w_{\text{dm}}, w_{\text{de}})$				
Data set	$w_{\text{dm}}$	$w_{\text{de}}$	$\chi^2_{\text{min}}$	$\chi^2_{\text{d.o.f.}}$
SNe Ia	$0.006^{+0.133}_{-0.096}$	$-1.003^{+0.12}_{-0.13}$	562.23	0.97
$(\mathcal{R}, l_A, z_*)$ CMB	$0.004 \pm 0.001$	$-1.197^{+0.057}_{-0.053}$	1.11	1.11
$H(z)$	$0.007^{+0.069}_{-0.058}$	$-1.197^{+0.14}_{-0.13}$	8.05	0.73
SNe + CMB + BAO + $H(z)$	$0.006 \pm 0.001$	$-1.115 \pm 0.033$	578.84	0.97

TABLE I. Best estimated values for  $(w_{\text{dm}}, w_{\text{de}})$ . See figures 1 and 2 for the confidence intervals. We find that the cosmological data used in the present work favor a non-vanishing magnitude, positive value for  $w_{\text{dm}}$  suggesting a *warm* dark matter, in addition to  $w_{\text{de}} < -1$  indicating a *phantom* dark energy. In order to compare these results with the  $\Lambda$ CDM model we computed the value of the  $\chi^2$  function evaluated at  $(w_{\text{dm}} = 0, w_{\text{de}} = -1)$  using the same four cosmological data sets (SNe + CMB + BAO +  $H(z)$ ) together, finding a value of  $\chi^2_{\Lambda\text{CDM}} = 740.5$ , that is clearly greater than  $\chi^2_{\text{min}} = 578.8$  obtained in the present work for  $w_{\text{dm}} = 0.006$ ,  $w_{\text{de}} = -1.115$ , indicating that the  $\Lambda$ CDM model fits not too well the cosmological data compared with the latter values. It was assumed  $\Omega_{\text{b}0} = 0.0458$ ,  $\Omega_{\text{r}0} = 0.0000758$ ,  $\Omega_{\text{dm}0} = 0.23$  and  $H_0 = 73.8$  km/(s·Mpc). The errors correspond to 68.3% of confidence level ( $1\sigma$ ).

Best estimates for $w_{\text{dm}}$			
$w_{\text{dm}}$	$w_{\text{de}}$	$\chi^2_{\text{min}}$	$\chi^2_{\text{d.o.f.}}$
$0.009 \pm 0.002$	-1	591.57	0.99
$0.006 \pm 0.002$	-1.1	579.02	0.97
$0.012 \pm 0.002$	-0.9	628.21	1.05

TABLE II. Best estimated values for  $w_{\text{dm}}$  when it is assumed the different values of  $w_{\text{de}} = -1, -1.1, -0.9$  for the dark energy. In the three cases it is found a non-vanishing positive value for  $w_{\text{dm}}$ . We find also that the best fit to data is for the case when it is assumed  $w_{\text{de}} = -1.1$ , i.e., it has the smallest value of  $\chi^2_{\text{min}}$  compared with the other two cases. It was used the joint SNe + CMB + BAO +  $H(z)$  data sets. The errors are at 99.73% of confidence level ( $3\sigma$ ). See figure 3 for the likelihood functions.

$$\chi^2_{\text{SNe}}(w_{\text{dm}}, H_0) \equiv \sum_{k=1}^n \left( \frac{\mu^t(z_k, w_{\text{dm}}, H_0) - \mu_k}{\sigma_k} \right)^2 \quad (14)$$

where  $\mu_k$  is the observed distance moduli of the k-th supernova, with a standard deviation of  $\sigma_k$  in its measurement, and  $n = 580$ . It was used a *constant* prior distribution function for  $H_0$  to marginalize it (i.e., it is not assumed any particular value of  $H_0$ ) because  $H_0$  is a nuisance parameter in the SNe Ia test.

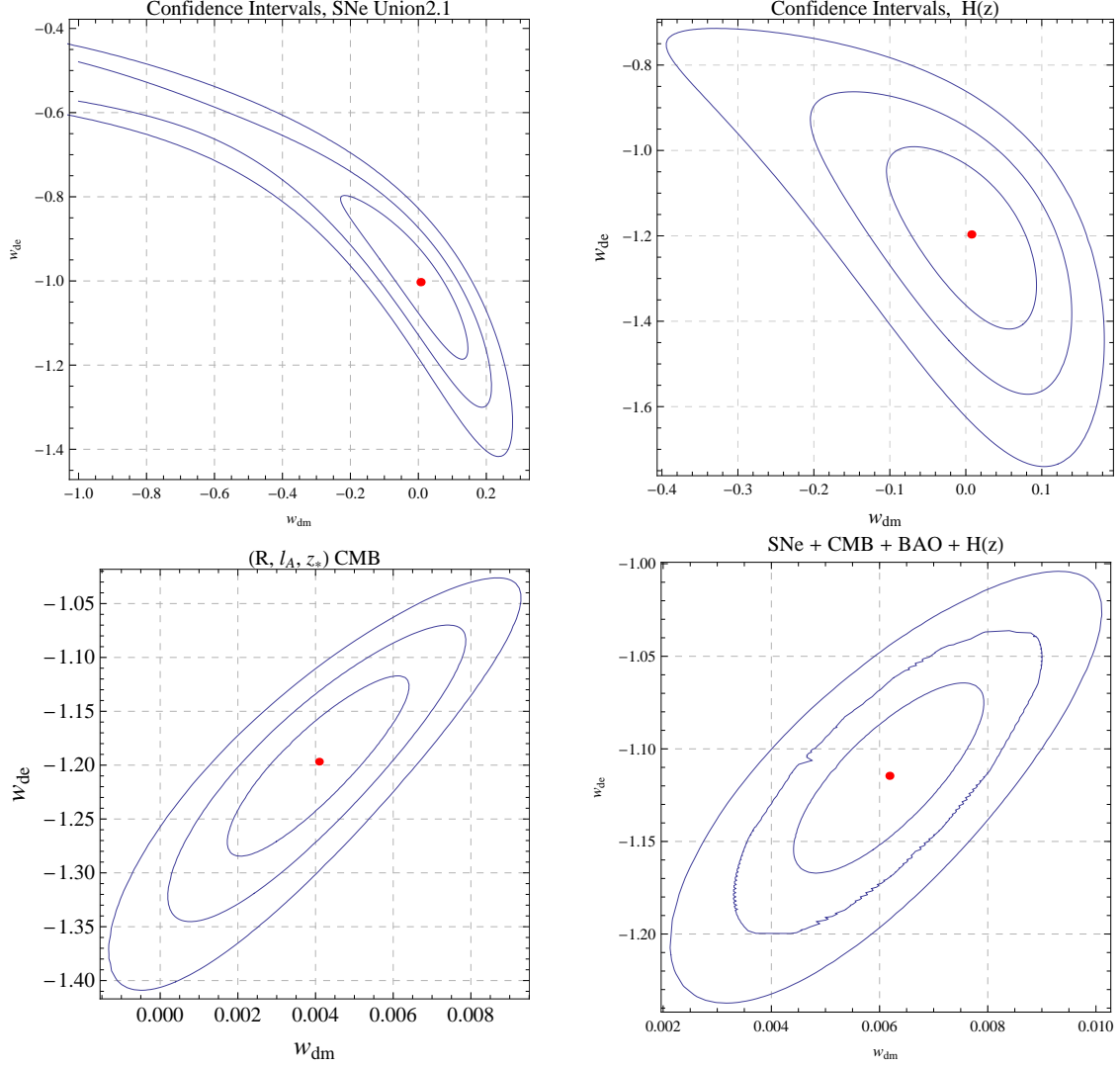


FIG. 1. Confidence intervals for  $(w_{\text{dm}}, w_{\text{de}})$ . The upper left panel corresponds to the use of the SNe Ia data set release (2012) “Union 2.1” of the SCP [22], through the minimization of the  $\chi^2$  function (14). The upper right panel corresponds to the use of Hubble parameter data at different redshifts using the  $\chi^2$  function (30). The lower left panel corresponds to the use of three observational data  $(\mathcal{R}, l_A, z_*)$  given by WMAP-7y [19], through the  $\chi^2$  function (22). And the lower right panel corresponds to the use of the total  $\chi^2$  function (31) that contains the four type of cosmological observations together SNe + CMB + BAO +  $H(z)$ . The best estimated values for  $(w_{\text{dm}}, w_{\text{de}})$  of each panel are indicated with the red point and the magnitudes are shown in table I. It is assumed a spatially flat FRW universe and for the baryon, dark matter, radiation parameter densities and the Hubble constant it was assumed the values of  $\Omega_{\text{b}0} = 0.0458$ ,  $\Omega_{\text{dm}0} = 0.23$ ,  $\Omega_{\text{r}0} = 0.0000766$  [19, 21] and  $H_0 = 73.8$  km/s-Mpc [18] respectively. The contour plots correspond to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level.

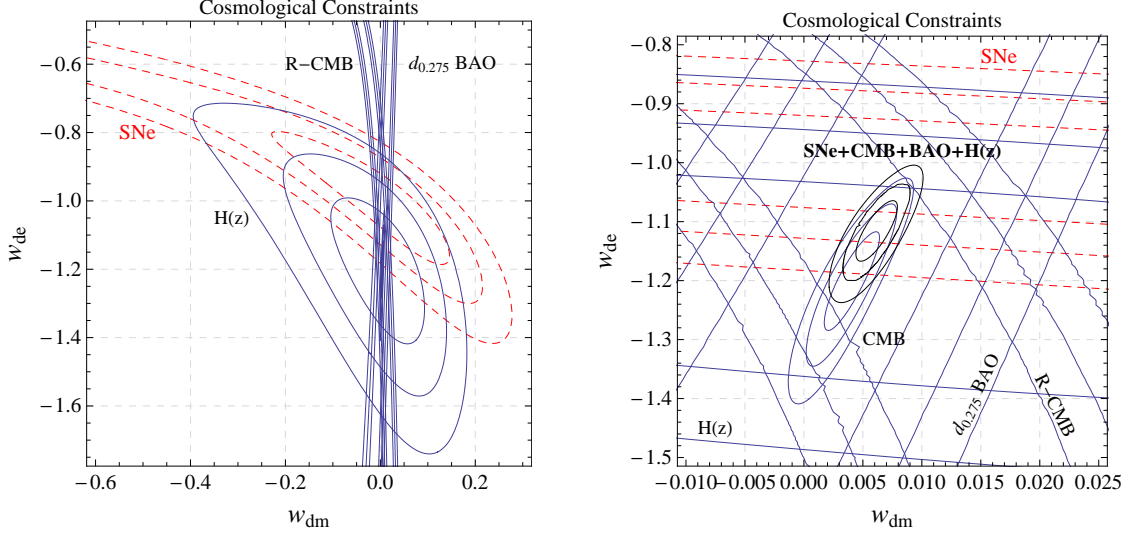


FIG. 2. Confidence intervals (CI) all together for  $(w_{\text{dm}}, w_{\text{de}})$  calculated with the different cosmological data sets (cf. figure 1 for CI separately for each cosmological data set). The CI labeled by “R-CMB” and “ $d_{0.275}$  BAO” come from the use of the shift parameter  $\mathcal{R}$  and the distance ratio  $d_z$  at  $z = 0.275$  of BAO computed through the  $\chi^2$  functions defined at (17) and (29) respectively. The right panel corresponds to a zoom in of the left panel in order to show the CI coming from the use of the  $(\mathcal{R}, l_A, z_*)$  CMB distance priors and the joint SNe+CMB+BAO+ $H(z)$ . The best estimated values are shown in table I. The interval regions corresponds to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level.

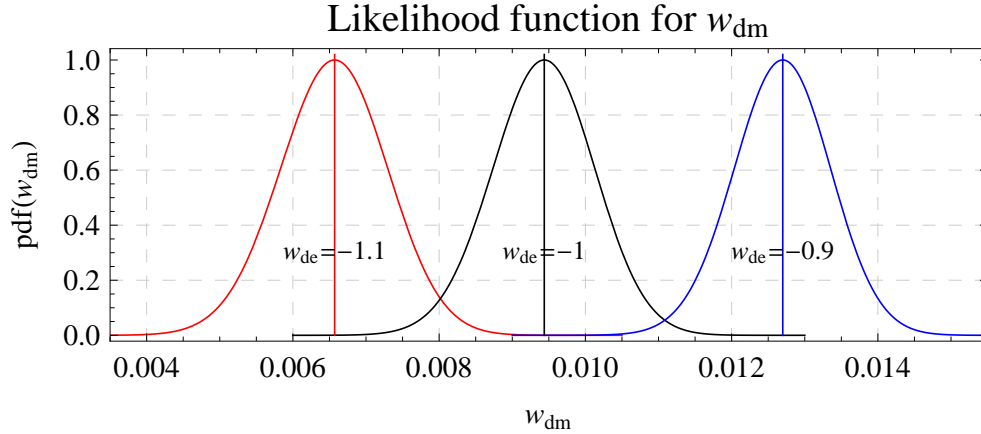


FIG. 3. Likelihood functions for  $w_{\text{dm}}$  when it is assumed the values of  $w_{\text{de}} = -1, -1.1, -0.9$  for the EoS of dark energy. See table II for the best estimated values of  $w_{\text{dm}}$ . It is assumed a spatially flat FRW universe and for the baryon, dark matter, radiation parameter densities and the Hubble constant it was assumed the values of  $\Omega_{\text{b}0} = 0.0458$ ,  $\Omega_{\text{dm}0} = 0.23$ ,  $\Omega_{\text{r}0} = 0.0000766$  and  $H_0 = 73.8$  km/s·Mpc respectively.



Best estimates for $(w_{\text{dm}}, \Omega_{\text{dm}0})$					
Data set	$w_{\text{dm}}$	$\Omega_{\text{dm}0}$	$w_{\text{de}}$	$\chi^2_{\text{min}}$	$\chi^2_{\text{d.o.f.}}$
SNe Ia	$0.004 \pm 0.27$	$0.229^{+0.16}_{-0.09}$	-1	562.22	0.972
	-0.103	0.316	-1.1	562.20	0.972
	0.177	0.138	-0.9	562.275	0.972
$(\mathcal{R}, l_A, z_*)$ CMB	$-0.0009 \pm 0.003$	$0.183^{+0.010}_{-0.009}$	-1	2.558	2.558
	0.001	0.206	-1.1	0.04	0.04
	-0.007	0.153	-0.9	10.90	10.90
$H(z)$	$0.215^{+0.226}_{-0.212}$	$0.114^{+0.076}_{-0.050}$	-1	8.271	0.751
	0.085	0.176	-1.1	8.12	0.738
	0.426	0.057	-0.9	8.55	0.777
SNe + CMB + BAO + $H(z)$	$0.005 \pm 0.002$	$0.204 \pm 0.008$	-1	582.11	0.978
	$0.005 \pm 0.001$	$0.223 \pm 0.008$	-1.1	578.27	0.971
	$0.005 \pm 0.002$	$0.184 \pm 0.007$	-0.9	601.41	1.010

TABLE III. Best estimated values of the parameter density of dark matter  $\Omega_{\text{dm}0}$  and the  $w_{\text{dm}}$  of the EoS of dark matter ( $p_{\text{dm}} = w_{\text{dm}} \cdot \rho_{\text{dm}}$ ). The first column shows the cosmological data sets used to compute the best estimates shown in second and third columns. The fourth column indicates the assumed value for  $w_{\text{de}}$ . The fifth and sixth columns correspond to the minimum value of the  $\chi^2$  function,  $\chi^2_{\text{min}}$ , and  $\chi^2$  by degrees of freedom,  $\chi^2_{\text{d.o.f.}}$ , respectively. The latter is defined as  $\chi^2_{\text{d.o.f.}} = \chi^2_{\text{min}}/(n-p)$ , where  $n$  is the number of data and  $p$  the number of free parameters (in this case  $p = 2$ ). The computed values come from the minimization of the  $\chi^2$  functions defined in (14), (22), (30) and (31) respectively. The fourth row (SNe + CMB + BAO +  $H(z)$ ) encloses the information of all the cosmological observations used in the present work to constrain the values of  $(\Omega_{\text{dm}0}, w_{\text{dm}})$ . Notice that  $w_{\text{dm}}$  has a positive value, favoring a *warm* instead of a cold dark matter. See figures 4 to 10 for the confidence intervals.

## 2. Cosmic Microwave Background Radiation

We use the WMAP 7-years distance priors shown in table 9 of [19], composed of the shift parameter  $\mathcal{R}$ , the acoustic scale  $l_A$  and the redshift of decoupling  $z_*$ .

The shift parameter  $\mathcal{R}$  is defined as

$$\mathcal{R} = \frac{H_0 \sqrt{\Omega_{\text{m}0}}}{c} (1 + z_*) D_A(z_*) \quad (15)$$

where  $\Omega_{\text{m}0}$  corresponds to the total present pressureless matter in the Universe, i.e.  $\Omega_{\text{m}0} = \Omega_{\text{b}0} + \Omega_{\text{dm}0}$ , and  $D_A$  is the proper angular diameter distance given by

$$D_A(z) = \frac{c}{(1+z)H_0} \int_0^z \frac{dz'}{E(z', w_{\text{dm}})}. \quad (16)$$

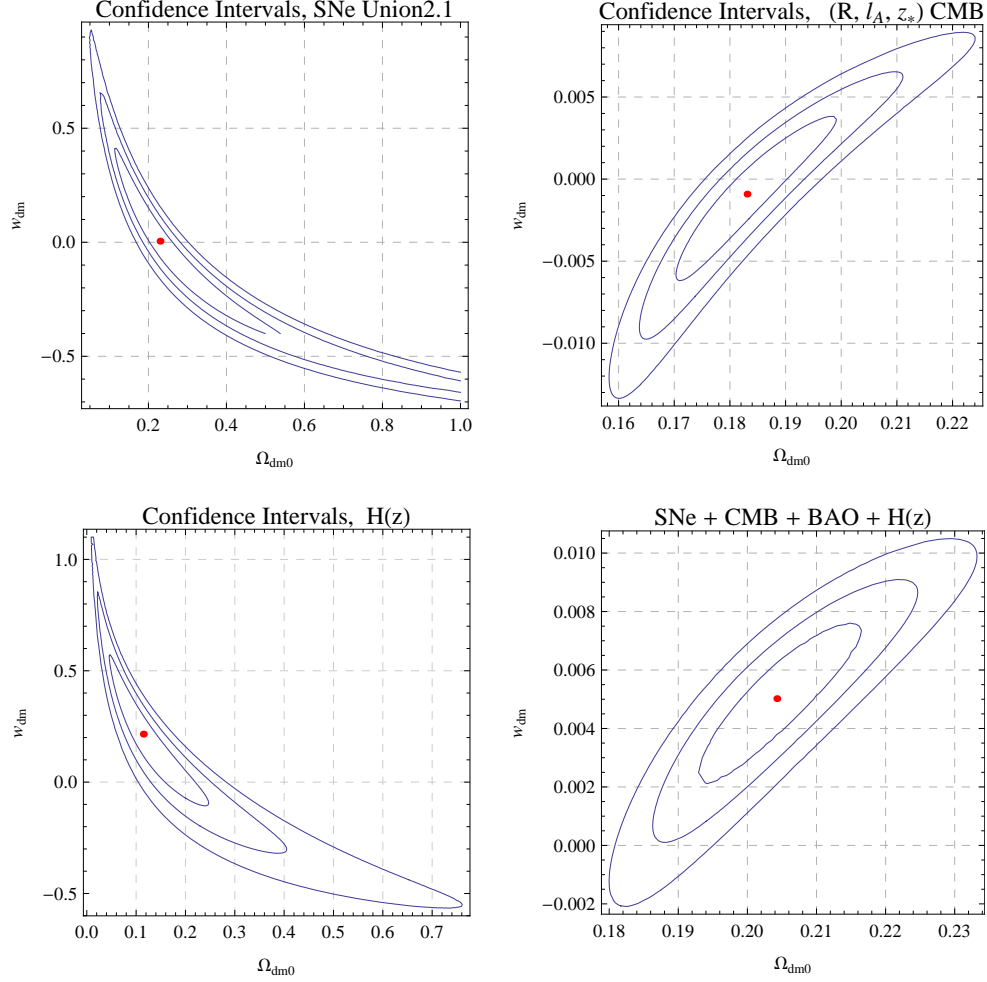


FIG. 4. Confidence intervals for the present-day value of the parameter density of dark matter  $\Omega_{\text{dm}0}$  versus the  $w_{\text{dm}}$  of the EoS of dark matter, and assuming  $w_{\text{de}} = -1$  for the dark energy. The upper left, right, lower left and right panels correspond to the use of SNe,  $(\mathcal{R}, l_A, z_*)$  of CMB,  $H(z)$  and the joint SNe+CMB+BAO+ $H(z)$  data sets respectively. The best estimated values are indicated with the red point and the magnitudes are shown in table III. It is assumed a spatially flat FRW universe and for the baryon, dark matter, radiation parameter densities and the Hubble constant it was assumed the values of  $\Omega_{\text{b}0} = 0.0458$ ,  $\Omega_{\text{dm}0} = 0.23$ ,  $\Omega_{\text{r}0} = 0.0000766$  and  $H_0 = 73.8$  km/s-Mpc respectively. The interval regions correspond to 1, 2 and  $3\sigma$  of confidence level.

for a spatially flat Universe. With  $\mathcal{R}$  we can defined a  $\chi^2$  function as

$$\chi_{\text{R-CMB}}^2(w_{\text{dm}}, H_0) \equiv \left( \frac{\mathcal{R} - \mathcal{R}_{\text{obs}}}{\sigma_{\mathcal{R}}} \right)^2 \quad (17)$$

where  $\mathcal{R}_{\text{obs}} = 1.725$  is the “observed” value of the shift parameter and  $\sigma_{\mathcal{R}} = 0.018$  the standard deviation of the measurement (cf. table 9 of [19]).

The acoustic scale  $l_A$  is defined as

$$l_A \equiv (1 + z_*) \frac{\pi D_A(z_*)}{r_s(z_*)}, \quad (18)$$

where  $r_s(z_*)$  corresponds to the comoving sound horizon at the decoupling epoch of photons,  $z_*$ , given by

$$r_s(z) = \frac{c}{\sqrt{3}} \int_0^{1/(1+z)} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_{b0}/4\Omega_{\gamma0})a}} \quad (19)$$

where as mentioned above, we use  $\Omega_{\gamma0} = 2.469 \times 10^{-5} h^{-2}$  as the present-day photon energy density parameter, and  $\Omega_{b0} = 0.02255 h^{-2}$  as the baryonic matter component, as reported by Komatsu et al. 2011 [19]. We compute the theoretical value of  $z_*$  from the fitting formula proposed by Hu and Sugiyama [23]

$$z_* = 1048 \left[ 1 + 0.00124(\Omega_{b0} h^2)^{-0.738} \right] \left[ 1 + g_1(\Omega_{m0} h^2)^{g_2} \right], \quad (20)$$

where

$$g_1 = \frac{0.0783(\Omega_{b0} h^2)^{-0.238}}{1 + 39.5(\Omega_{b0} h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_{b0} h^2)^{1.81}}. \quad (21)$$

The  $\chi^2$  function using the three distance priors  $(l_A, \mathcal{R}, z_*)$  is defined as

$$\chi_{\text{CMB}}^2(w_{\text{dm}}, H_0) = \sum_{i,j=1}^3 (x_i - d_i)(C^{-1})_{ij}(x_j - d_j) \quad (22)$$

where  $x_i \equiv (l_A, \mathcal{R}, z_*)$  are the theoretical values predicted by the model and  $d_i \equiv (l_A = 302.09, \mathcal{R} = 1.725, z_* = 1091.3)$  are the observed ones. For  $H_0$  it was assumed the latest reported value of  $H_0 = 73.8 \text{ km/s/Mpc}$  [18]. The  $C_{ij}^{-1}$  is the inverse covariance matrix with entries [19]

$$C^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.27 & -113.180 \\ 1.333 & 113.180 & 3.414 \end{pmatrix} \quad (23)$$

### 3. Baryon Acoustic Oscillations

We use the baryon acoustic oscillation (BAO) data from the SDSS 7-years release [24], expressed in terms of the distance ratio  $d_z$  at  $z = 0.275$  defined as

$$d_{0.275} \equiv \frac{r_s(z_d)}{D_V(0.275)} \quad (24)$$

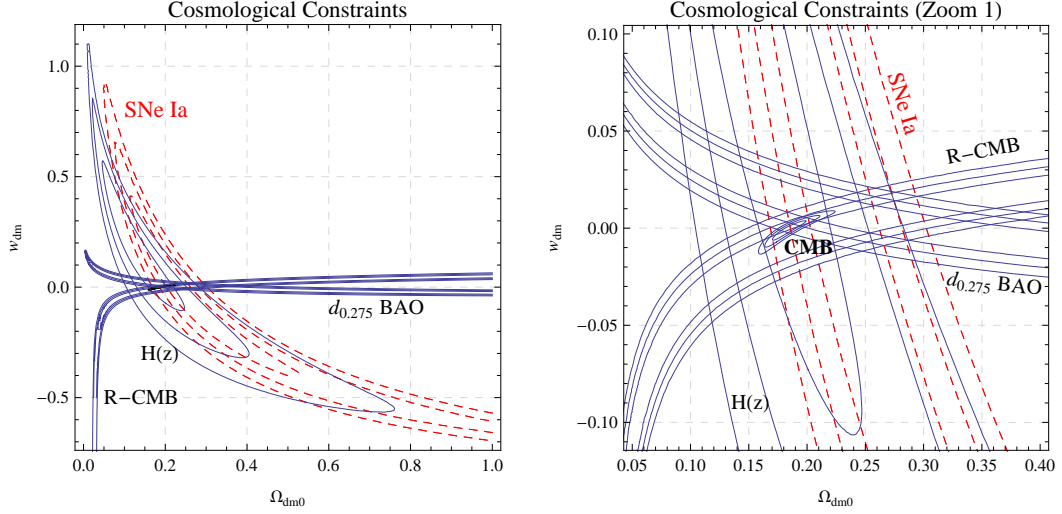


FIG. 5. Confidence intervals (CI) for  $(\Omega_{\text{dm}0}, w_{\text{dm}})$ , calculated with the different cosmological data sets (see figure 4). It is assumed a value of  $w_{\text{de}} = -1$  for the parameter of EoS of dark energy. The right panel corresponds to a “zoom in” of the left panel in order to show the tiny CI that come from the use of the  $(\mathcal{R}, l_A, z_*)$  CMB distance priors through the  $\chi^2_{\text{CMB}}$  function (22) and labeled as “CMB”. The CI from the total  $\chi^2$  function (31) are even smaller than those of the CMB and so they are shown in figure 6. The best estimated values are shown in table III. The interval regions corresponds to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level.

where  $z_d$  is the redshift at the baryon drag epoch computed from the fitting formula [25]

$$z_d = 1291 \frac{(\Omega_{\text{m}0} h^2)^{0.251}}{1 + 0.659(\Omega_{\text{m}0} h^2)^{0.828}} \left[ 1 + b_1 (\Omega_{\text{m}0} h^2)^{b_2} \right], \quad (25)$$

$$b_1 = 0.313(\Omega_{\text{m}0} h^2)^{-0.419} \left[ 1 + 0.607(\Omega_{\text{m}0} h^2)^{0.674} \right], \quad (26)$$

$$b_2 = 0.238(\Omega_{\text{m}0} h^2)^{0.223}. \quad (27)$$

For a flat Universe,  $D_V(z)$  is defined as

$$D_V(z) = c \left[ \left( \int_0^z \frac{dz'}{H(z')} \right)^2 \frac{z}{H(z)} \right]^{1/3}. \quad (28)$$

It contains the information of the visual distortion of a spherical object due the non-Euclidianity of the FRW spacetime.

The value  $d_{0.275}^{\text{obs}}$  contains the information of the other two pivots,  $d_{0.2}$  and  $d_{0.35}$ , usually used by other authors, with a precision of 0.04% [24].

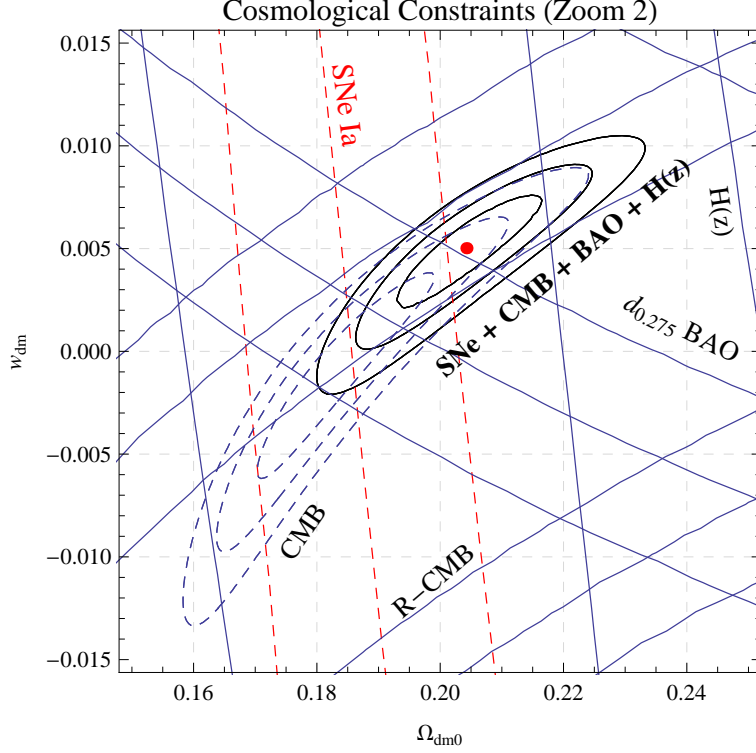


FIG. 6. Confidence intervals for  $(\Omega_{\text{dm}0}, w_{\text{dm}})$ . This figure corresponds to a “zoom in” of the figures 4 and 5 to show the CI that come from the use of all the observational data sets together through the total  $\chi^2$  function defined in (31) and labeled as “SNe + CMB + BAO +  $H(z)$ ”. The best estimated values computed with the total  $\chi^2$  function are  $w_{\text{dm}} = 0.005$ ,  $\Omega_{\text{dm}0} = 0.204$  (see table III). The intervals regions correspond to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level. Notice that  $w_{\text{dm}} > 0$  with 95% confidence level, suggesting a *warm* dark matter.

The  $\chi^2$  function for BAO is defined as

$$\chi_{\text{BAO}}^2(w_{\text{dm}}, H_0) \equiv \left( \frac{d_{0.275} - d_{0.275}^{\text{obs}}}{\sigma_d} \right)^2 \quad (29)$$

where  $d_{0.275}^{\text{obs}} = 0.139$  is the observed value and  $\sigma_d = 0.0037$  the standard deviation of the measurement [24]. For  $H_0$  it was assumed the latest reported value of  $H_0 = 73.8$  km/s/Mpc [18].

#### 4. Hubble expansion rate

For the Hubble parameter, we use the 13 available data, 11 data come from the table 2 of Stern et al. (2010) [26] and the two following data come from Gaztanaga et al. 2010 [27]:  $H(z = 0.24) = 79.69 \pm 2.32$  and  $H(z = 0.43) = 86.45 \pm 3.27$  km/s/Mpc. For the present value of the Hubble parameter, we take the value reported by Riess et al 2011 [18]:  $H(z = 0) \equiv H_0 = 73.8 \pm 2.4$

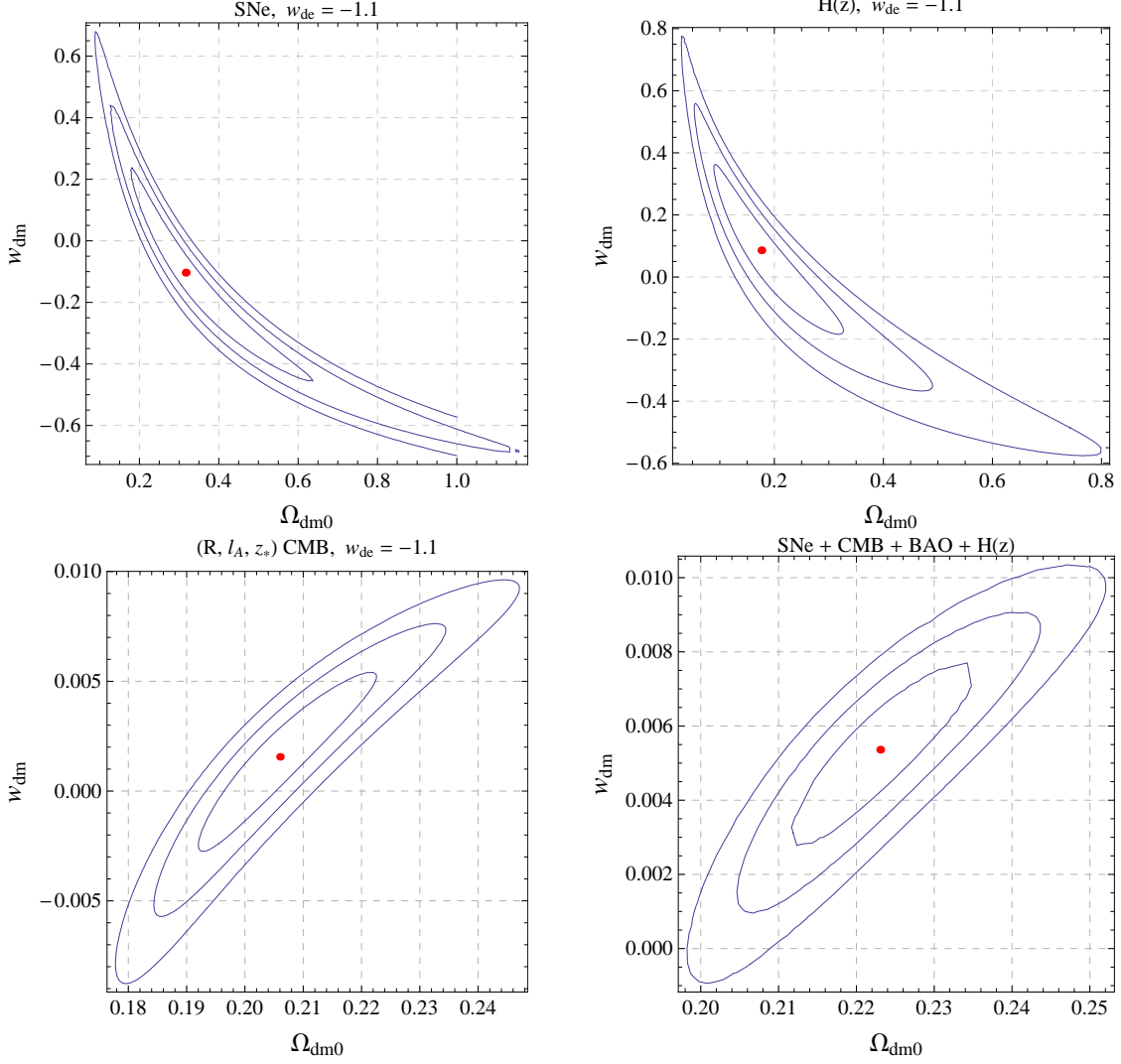


FIG. 7. Confidence intervals for  $(\Omega_{\text{dm}0}, w_{\text{dm}})$  when it is assumed the value of  $w_{\text{de}} = -1.1$  for the parameter of EoS of dark energy, i.e., a phantom dark energy. See table III for the values of the best estimates. The interval regions corresponds to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level.

km/s/Mpc. The  $\chi^2$  function is defined as

$$\chi_{\text{H}}^2(w_{\text{dm}}, H_0) = \sum_i^{13} \left( \frac{H(z_i, w_{\text{dm}}) - H_i^{\text{obs}}}{\sigma_H} \right)^2 \quad (30)$$

where  $H(z_i)$  is the theoretical value predicted by the model and  $H_i^{\text{obs}}$  is the observed value with its standard deviation  $\sigma_H$ .

Finally, with the  $\chi^2$  functions defined above we construct the total  $\chi^2$  function given by

$$\chi^2 = \chi_{\text{SNe}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{H}}^2. \quad (31)$$

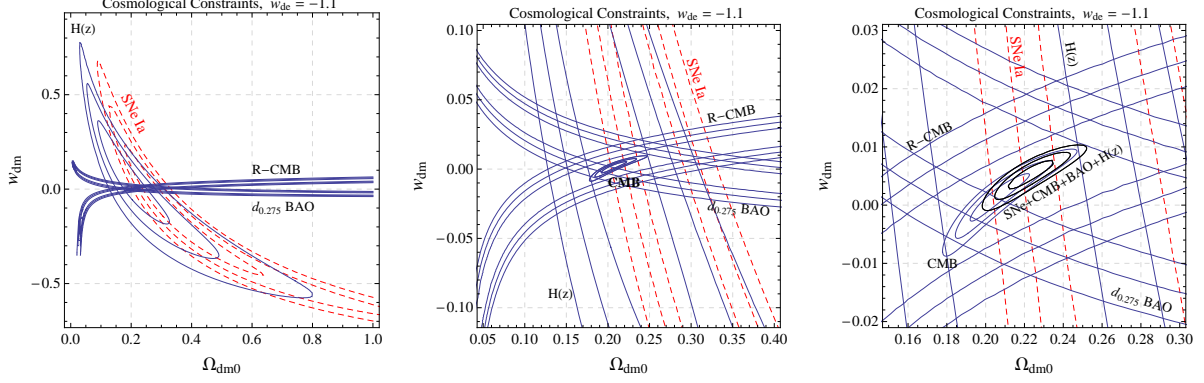


FIG. 8. Confidence intervals for  $(\Omega_{\text{dm}0}, w_{\text{dm}})$  when it is assumed the value of  $w_{\text{de}} = -1.1$  for the parameter of EoS of dark energy, i.e., a phantom dark energy. See also figure 7. Table III shows the values of the best estimates for this case. The central and right panels correspond to a zoom in of the left panel. The interval regions corresponds to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level.

Age of the Universe						
Age (Gyr)	$w_{\text{dm}}$	$w_{\text{de}}$	$\Omega_{\text{dm}0}$	Table	Point	
$13.23 \pm 0.06$	0.006	-1.115	0.23	I	A	
$12.97 \pm 0.012$	0.009	-1	0.23	II	B	
$13.19 \pm 0.015$	0.007	-1.1	0.23	II	C	
$12.7 \pm 0.01$	0.013	-0.9	0.23	II	D	
$13.37 \pm 0.14$	0.005	-1	0.204	III	E	
$13.31 \pm 0.12$	0.005	-1.1	0.223	III	F	
$13.4 \pm 0.12$	0.005	-0.9	0.184	III	G	

TABLE IV. Age of the Universe given in gigayears (first column) when it is assumed certain values for  $(w_{\text{dm}}, w_{\text{de}}, \Omega_{\text{dm}0})$  shown in the 2nd to 4th columns and that comes from the best estimates of  $w_{\text{dm}}$  shown in tables I–III (fifth column). The last column indicates the letters used in figure 11 to label those points.  $H_0$  is assumed to be 73.8 km/s/Mpc.

We minimize this function with respect to the set of parameters  $(w_{\text{dm}}, w_{\text{de}})$ ,  $(w_{\text{dm}}, \Omega_{\text{dm}0})$  and  $w_{\text{dm}}$  alone, to compute their best estimated values and confidence intervals or likelihood functions.

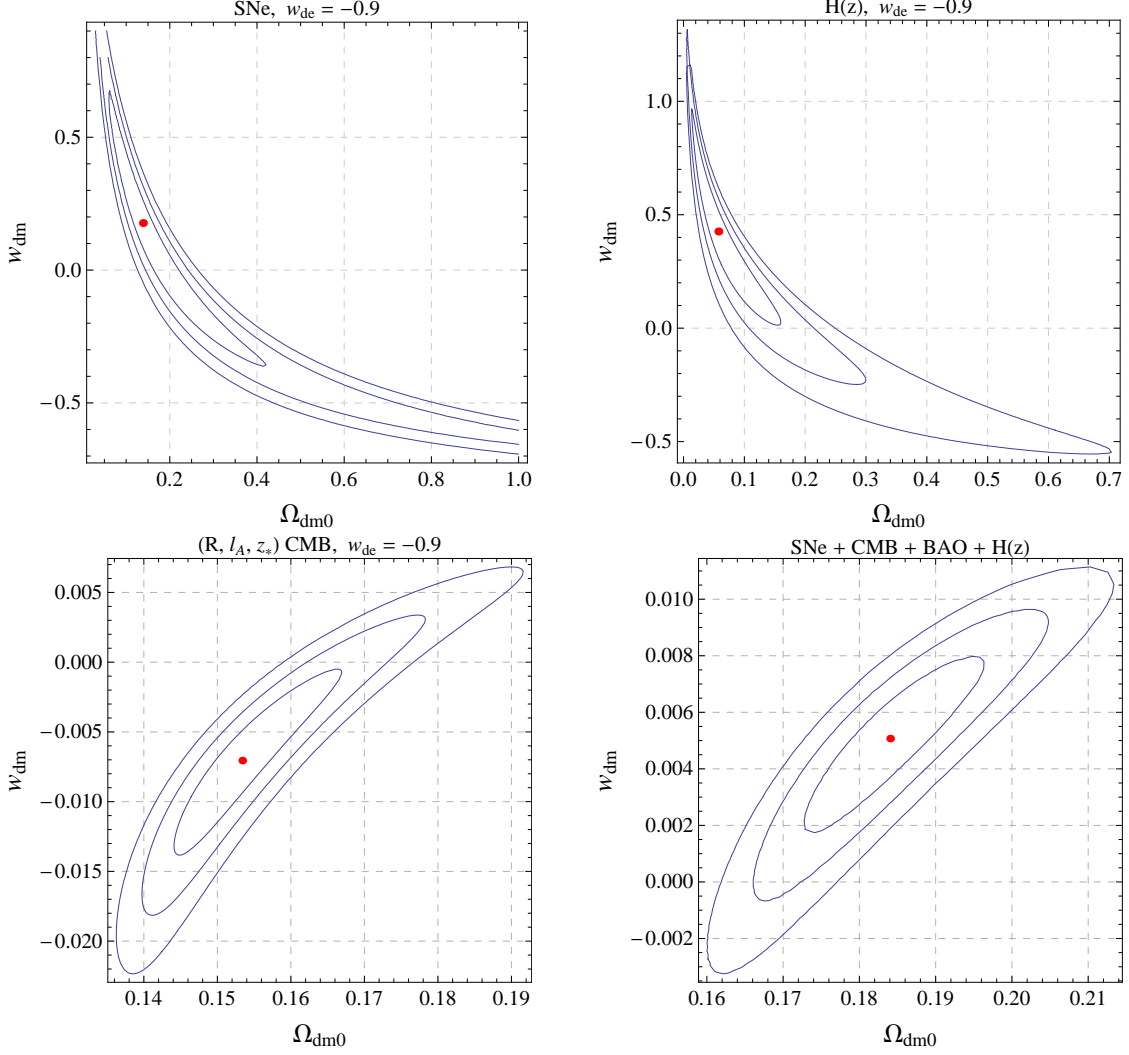


FIG. 9. Confidence intervals for  $(\Omega_{\text{dm}0}, w_{\text{dm}})$  when it is assumed the value of  $w_{\text{de}} = -0.9$  for the parameter of EoS of dark energy. See table III for the values of the best estimates. The interval regions corresponds to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level.

### 5. The age of the Universe

Using the fact that  $H = \dot{a}/a$ , we can rewrite the eq. (8) as an ordinary differential equation (ODE) for the scale factor  $a$  in terms of the cosmic time as

$$\frac{da}{dt} - \beta H_0 a \sqrt{\frac{\Omega_{\text{r}0}}{a^4} + \frac{\Omega_{\text{b}0}}{a^3} + \frac{\Omega_{\text{de}0}}{a^{3(1+w_{\text{de}})}} + \frac{\Omega_{\text{dm}0}}{a^{3(1+w_{\text{dm}})}}} = 0, \quad (32)$$

where  $\beta = 1.022729 \times 10^{-3}$  is introduced to give the units of time in *gigayears* (Gyr) when the value of the Hubble constant is given in units of km/(s·Mpc). For the conversion of units, we use the values of 1 year = 31558149.8 seconds (a sidereal year)[29] and 1 Mpc =  $3.0856776 \times 10^{19}$  km



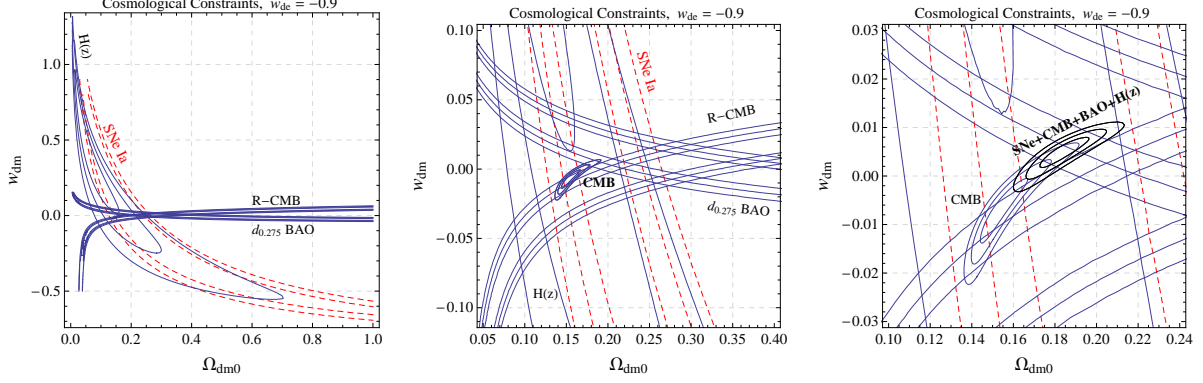


FIG. 10. Confidence intervals for  $(\Omega_{\text{dm}0}, w_{\text{dm}})$  when it is assumed the value of  $w_{\text{de}} = -0.9$  for the parameter of EoS of dark energy. See also figure 9. Table III shows the values of the best estimates for this case. The central and right panels correspond to a zoom in of the left one. The interval regions corresponds to 68.3% ( $1\sigma$ ), 95.4% ( $2\sigma$ ) and 99.73% ( $3\sigma$ ) of confidence level.

[29], so  $\beta = (31558149.8 \times 10^9)/3.0856776$ .

We solve numerically the ODE (32) with the initial condition  $a(t=0) = 0$  [30] and compute the value  $t_{\text{today}}$  of the age of the Universe through the condition  $a(t_{\text{today}}) = 1$ .

Evaluating the numerical solution of the ODE (32) at the best estimates and assuming the values of  $H_0 = 73.8 \pm 2.4$  [18],  $\Omega_{\text{r}0} = 0.0000758$ ,  $\Omega_{\text{b}0} = 0.0458 \pm 0.0016$  [19] we find an age of the Universe. See table IV and figure 11. From the oldest globular clusters the age of the Universe is constrained to  $12.9 \pm 2.9$  Gyr [28].

#### IV. DISCUSSION AND CONCLUSIONS

We explored the constraints on the value of the parameter  $w_{\text{dm}}$  of the barotropic EoS of the dark matter to investigate the “warmness” of the dark matter fluid. The model is composed by the dark matter and dark energy fluids in addition to the radiation and baryon components. We constrained the value of  $w_{\text{dm}}$  using the SNe Ia “Union 2.1” of the SCP data set, the three observational  $(\mathcal{R}, l_A, z_*)$  data from the CMB given by WMAP-7y, the distance ratio  $d_z$  at  $z = 0.275$  of BAO and the Hubble parameter data at different redshifts.

We calculated the best estimated values for the pair of parameters  $(w_{\text{dm}}, w_{\text{de}})$ ,  $(w_{\text{dm}}, \Omega_{\text{dm}0})$  and also  $w_{\text{dm}}$  alone, where  $w_{\text{de}}$  and  $\Omega_{\text{dm}0}$  are the parameter of the barotropic EoS of dark energy and the present-day value of the density parameter of dark matter respectively.

When  $w_{\text{dm}}$  is estimated together with  $w_{\text{de}}$  we found that the cosmological data prefer the

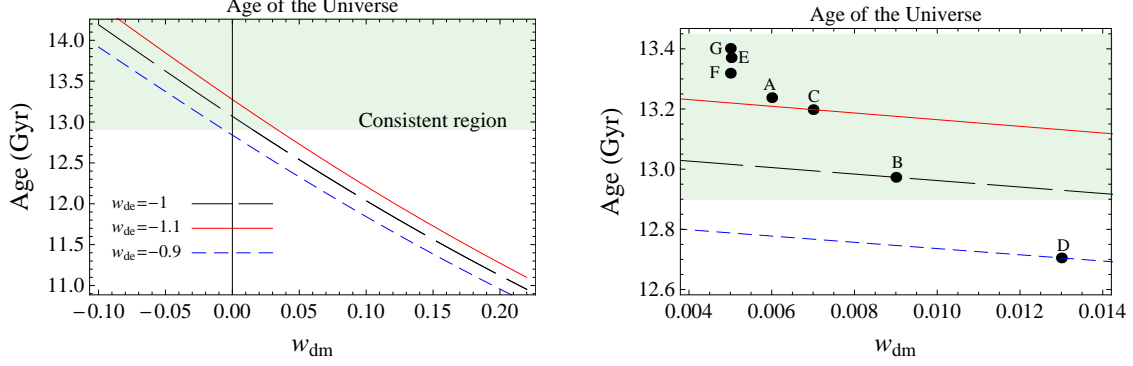


FIG. 11. Age of the Universe given in gigayears (Gyr) as a function of  $w_{\text{dm}}$ . The black long dashed, the solid red and the blue short dashed lines correspond to assume the values of  $w_{\text{de}} = -1, -1.1, -0.9$  respectively. The right panel corresponds to a zoom in of the left one, where the points locate the inferred value of the age of the Universe when the eq. (32) is evaluated at the best estimated values for  $w_{\text{dm}}$  (see section III 5 and table IV). The letters that label the points correspond to the values of  $(w_{\text{dm}}, \text{Age}, w_{\text{de}}, \Omega_{\text{dm}0})$  where: A =  $(0.006, 13.23, -1.115, 0.23)$ , B =  $(0.009, 12.97, -1, 0.23)$ , C =  $(0.007, 13.19, -1.1, 0.23)$ , D =  $(0.013, 12.7, -0.9, 0.23)$ , E =  $(0.005, 13.37, -1, 0.204)$ , F =  $(0.005, 13.31, -1.1, 0.223)$  and G =  $(0.005, 13.4, -0.9, 0.184)$ . The shaded area corresponds to the consistent region for the age of the Universe estimated from the oldest globular clusters (Age =  $12.9 \pm 2.9$  Gyr [28]).

value of  $w_{\text{dm}} = 0.006 \pm 0.001$ , suggesting a *warm* dark matter, and  $w_{\text{de}} = -1.11 \pm 0.03$  that corresponds to a phantom dark energy, instead a cold dark matter and a cosmological constant ( $w_{\text{dm}} = 0, w_{\text{de}} = -1$ ). See table I and figures 1 and 2.

In order to study the dependence of the estimated value for  $w_{\text{dm}}$  with respect to the value of  $w_{\text{de}}$  of the dark energy, we computed the best estimate of  $w_{\text{dm}}$  as the only free parameter but assuming three different values of  $w_{\text{de}} = -1, -1.1, -0.9$ . We found the values of  $w_{\text{dm}} = 0.009 \pm 0.002, 0.006 \pm 0.002, 0.012 \pm 0.002$  when it is assumed the values of  $w_{\text{de}} = -1, -1.1, -0.9$  respectively, where the errors were computed at  $3\sigma$  (99.73%), so, we found that  $w_{\text{dm}} > 0$  with at least 99.73% of confidence level. Additionally, from these three cases, the assumption of  $w_{\text{de}} = -1.1$  is the case that allows to fit better the model to data compared with the other two cases (see table II and figure 3).

When  $w_{\text{dm}}$  is constrained together with  $\Omega_{\text{dm}0}$  we found that the best fit to data is for ( $w_{\text{dm}} = 0.005 \pm 0.001, \Omega_{\text{dm}0} = 0.223 \pm 0.008$ ) and with the assumption of  $w_{\text{de}} = -1.1$ , instead of a cosmological constant (i.e.,  $w_{\text{de}} = -1$ ). We found also interesting to notice that the best estimated value of  $w_{\text{dm}}$  using all the combined data sets give the same value of  $w_{\text{dm}} = 0.005$  independent of the assumed value for  $w_{\text{de}}$ , where the three cases were  $w_{\text{de}} = -1, -1.1, -0.9$  (see the three rows at

the bottom of table III).

In all cases the best fit to data, measured through the  $\chi^2_{\text{d.o.f.}}$  magnitude, of the cosmological observations separately or all together (the joint SNe + CMB + BAO +  $H(z)$  data) correspond to the case when it is assumed  $w_{\text{de}} = -1.1$  (phantom dark energy) instead of a cosmological constant ( $w_{\text{de}} = -1$ ) or  $w_{\text{de}} = -0.9$ .

For the age of the Universe, we found a consistent value for the age when it is evaluated at the best estimated values for  $w_{\text{dm}}$ , except for the case when it is assumed  $w_{\text{de}} = -0.9$ . See table IV and figure 11.

On the other hand, Muller [13] and more recently Calabrese et al. [31] investigated the constraints on  $w_{\text{dm}}$  at perturbative level comparing with the large scale structure data and CMB anisotropies. They found the constraints  $-0.008 < w_{\text{dm}} < 0.0018$  and  $-0.0133 < w_{\text{dm}} < 0.0082$  respectively. We find that our results are comparable and consistent with these ones.

In summary, we found an evidence of a non-vanishing value  $w_{\text{dm}}$ . From the cosmological observations we found constraints on the values of  $w_{\text{dm}}$  around  $0.005 < w_{\text{dm}} < 0.01$  suggesting a *warm* dark matter, independent of assumed value for  $w_{\text{de}}$ , but where a value  $w_{\text{de}} < -1$  is preferred by the observations instead of the  $\Lambda$ CDM model. Our constraints on  $w_{\text{dm}}$  are consistent with perturbative analysis done in previous works.

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